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R^4 terms in supergravity and M-theory

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Abstract

Higher-order invariants and their rôle as possible counterterms for supergravity theories are reviewed. It is argued that $N = 8$ supergravity will diverge at 5 loops. The construction of R^4 superinvariants in string theory and M-theory is discussed.

1 Introduction

The title of this contribution to the Deserfest is an appropriate one in view of the fact that the first paper on three-loop (R^4) counterterms in supergravity was written by Stanley Deser, together with John Kay and Kelly Stelle, in 1979 [1]. It will be recalled that supergravity had been shown to be on-shell finite at one and two loops [2] and there were hopes that this would persist at higher loop orders, but these were somewhat spoiled by the above paper, on $N = 1$ supergravity, and by a follow-up which reported a similar result for $N = 2$ [3]. Any residual hopes that the maximally supersymmetric $N = 8$ supergravity theory in four dimensions would have special properties were removed by the observation that counterterms can easily be constructed as full superspace integrals, at seven loops in the linearised theory, and at eight loops if one wishes to preserve all the symmetries of the full non-linear theory including E_7 [4]. Subsequently, a linearised three-loop $N = 8$ invariant was also found [5]. In view of the difficulty of carrying out high loop calculations in quantum gravity, and of the success of string theory and M-theory, the subject has receded into the background over the years but interest in it has recently been reawakened by the development of new calculational techniques [6]. The implications of this work and its relation to the predictions of superspace power-counting arguments, both in conventional superspace and in harmonic superspace, will be the subject of the next section of the paper.

Notwithstanding these recent developments, R^4 and other higher order supersymmetric invariants are nowadays of most interest in the context of effective field theory actions in string theory and M-theory. Such terms could be interesting both theoretically, especially in the case of M-theory, and in applications to solutions and to beyond leading-order tests of various implications of duality. For such applications one would ideally like to know the complete bosonic part of the effective action. However, these terms seem to be extremely difficult to calculate systematically as will be seen below. Some partial results obtained from string theory and supersymmetry will be outlined in section 3 and the final section of the paper will be devoted to a brief exposition of a superspace approach to the problem in eleven dimensions [7]. This work seems to indicate that there is a unique invariant at this order which is supersymmetric and which is compatible with the M-theoretic input of five-brane anomaly cancellation.

2 Supergravity counterterms

The $D = 4, N = 8$ three-loop counterterm was first constructed in [5]. A manifestly supersymmetric and $SU(8)$ invariant version was given in [8] as an example of a superaction - an integral over superspace which involves integrating over fewer than the total number of odd coordinates. This type of integration, using conventional superspace and measures which carry $SU(N)$ representations, turns out to be equivalent to integration in certain harmonic superspaces where the number of odd coordinates is reduced [9]. The three-loop counterterm has a very simple form in this language.

In $D = 4$ N-extended supersymmetry harmonic superspaces are obtained from Minkowski superspace by adjoining to the latter a coset space of the internal $SU(N)$ symmetry group, chosen to be complex [10]. This coset can be thought of as parametrising sets of mutually anticom-

muting covariant derivatives (Ds and $\bar{D}s$). The simplest example is in $N = 2$ where we can select one D_α and one $\bar{D}_{\dot{\alpha}}$ to anticommute[1], and the ways this can be done are parametrised by the two-sphere $\mathbb{CP}^1$. For higher values of N there are more ways of choosing such sets of derivatives and therefore many different types of Grassmann analyticity (or generalised chirality) constraints that one can impose on superfields. In order for covariance with respect to the R-symmetry group to be maintained such G-analytic superfields must be allowed to depend on the coordinates of the coset space. It turns out that the G-analyticity constraints are compatible with ordinary (harmonic) analyticity on the coset space and that many field strength superfields can be described by superfields which are analytic in both senses. Harmonic analyticity ensures that such fields have short harmonic expansions.

To be more explicit, let $i = 1 \ldots N$ and $I = 1 \ldots N$ denote internal indices which are to be acted on by $SU(N)$ and the isotropy group respectively. We split I into three, $I = (r, R, r')$, where the ranges $1 \ldots p$ and $N-(q+1) \ldots N$ of r and r' cannot be greater than $N/2$; we write $u \in SU(N)$ as $u_I{}^i = (u_r{}^i, u_R{}^i, u_{r'}{}^i)$ and similarly for the inverse element $(u^{-1})_i{}^I$. This splitting is clearly preserved by the isotropy group $S(U(p) \times U(N-(p+q)) \times U(q))$, the coset space defined by this group being the flag space $\mathbb{F}_{p,N-q}$ of p-planes within $(N-q)$-planes in $\mathbb{C}^N$. Let

$$D_{\alpha I} = u_I{}^i D_{\alpha i}; \qquad \bar{D}^I_{\dot{\alpha}} = (u^{-1})_i{}^I \bar{D}^i_{\dot{\alpha}} \; , \tag{1}$$

then the derivatives $D_{\alpha r}$ and $\bar{D}^{r'}_{\dot{\alpha}}$ are mutually anti-commuting. Superfields which are annihilated by these derivatives are said to be G-analytic of type (p, q); the associated harmonic superspace is known as (N, p, q) harmonic superspace.

This formalism can be applied to $N = 8$ supergravity. The supergravity multiplet is described by the linearised field strength W_{ijkl}, $i = 1 \ldots 8$. The superfield W_{ijkl} is totally antisymmetric and transforms under the seventy-dimensional real representation of $SU(8)$. It obeys the constraints

$$\begin{aligned} \bar{W}^{ijkl} &= \frac{1}{4!}\epsilon^{ijklmnpq}W_{mnpq} && (2)\\ D_{\alpha i}W_{jklm} &= D_{\alpha[i}W_{jklm]} && (3)\\ \bar{D}^i_{\dot{\alpha}}W_{jklm} &= -\frac{4}{5}\delta^i_{[j}\bar{D}^n_{\dot{\alpha}}W_{klm]n} && (4) \end{aligned}$$

the third of which follows from the other two. Note that this superfield defines an ultra-short superconformal multiplet.

The same multiplet can be described by an analytic superfield W on $(8, 4, 4)$ harmonic superspace:

$$W := \epsilon^{rstu}u_r{}^i u_s{}^j u_t{}^k u_s{}^l W_{ijkl} \tag{5}$$

It is not difficult to see that the constraints

$$D_{\alpha r}W = \bar{D}^{r'}_{\dot{\alpha}}W = 0, \qquad r = 1 \ldots 4,\ r' = 5, \ldots 8\,, \tag{6}$$

[1] α and $\dot{\alpha}$ denote two-component spinor indices

together with analyticity on the internal coset, are equivalent to the differential constraints above, while the reality condition can also be formulated in harmonic superspace.

Since W depends on only half of the odd coordinates we can integrate it over an appropriate harmonic superspace measure $d\mu_{4,4} := d^4x\, du\, d^8\theta\, d^8\bar{\theta}$, where du is the usual Haar measure on the coset and where the odd variables are $\theta^{\alpha r'} := \theta^{\alpha i}(u^{-1})_i{}^{r'}$ and $\bar{\theta}^{\dot{\alpha}}_r := u_r{}^i \bar{\theta}^{\dot{\alpha}}_i$. In order to obtain an invariant the integrand must be $(4,4)$ G-analytic and must have the right charge with respect to the $U(1)$ subgroup of the isotropy group $S(U(4)\times U(4)) \sim U(1)\times SU(4)\times SU(4)$. The only possible integrand with these properties which can be constructed from W is W^4; it gives the harmonic superspace version of the three-loop counterterm in the form [9]

$$I_{3-loop} = \int d\mu_{4,4}\, W^4\,. \tag{7}$$

Since $W \sim \theta^4 C_{\alpha\beta\gamma\delta} + \bar{\theta}^4 \bar{C}_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} + \ldots$, where C is the Weyl spinor, it is apparent that integrating over the odd variables will give the square of the Bel-Robinson tensor.

All other possible invariants in linearised $N=8$ supergravity which are invariant under $SU(8)$ and which involve integrating over superspaces of smaller odd dimension than conventional superspace were recently classified [11]. There are just two, one in $(8,2,2)$ and one in $(8,1,1)$ harmonic superspace. They both have the same schematic form as the three-loop invariant but the measures and the definition of the superfield W differ. They correspond to five-loop and six-loop counterterms respectively:

$$\begin{aligned} I_{5-loop} &= \int d\mu_{2,2}\, W^4 \sim \int d^4x\, \partial^4\, R^4 + \ldots \\ I_{6-loop} &= \int d\mu_{1,1}\, W^4 \sim \int d^4x\, \partial^6\, R^4 + \ldots\ . \end{aligned} \tag{8}$$

In order to discuss the relevance of these counterterms to possible divergences in quantum supergravity one has to know how much supersymmetry can be preserved in the quantum theory [12, 13]. This means that we have to know the maximum number $M \leq 8$ of supersymmetries that can be realised linearly in the off-shell theory. If we stick to standard off-shell realisations, then only $M=4$ is allowed for $N=8$ supergravity [14, 15]. This would then suggest that counterterms should be integrals over sixteen odd coordinates leading to a prediction of the first divergence occurring at three loops. However, recent work by Bern *et al* has indicated that the coefficient of the three-loop counterterm is not divergent [6].

Some light can be shed on this apparent discrepancy by looking at the maximally supersymmetric $N=4$ Yang-Mills theory which becomes non-renormalisable in higher spacetime dimensions. In this case, Bern *et al* [6] also found improved UV behaviour compared to the predictions of superspace power-counting which were again based on the assumption that the best one can achieve off-shell is to preserve half of the total number of supersymmetries [16]. However, in the Yang-Mills case we know that there is an off-shell version with $N=3$ supersymmetry available in harmonic superspace [17]. If the superspace power-counting is adjusted for this, then the new predictions precisely match the calculational results [18].

This line of reasoning suggests that there should be an off-shell version of $N = 8$ supergravity with $M = 6$ supersymmetry which would correspond to the first divergence appearing at five loops. Results obtained in higher dimensions lend strong support to this contention. In [6] it was shown that the maximal supergravity theory in $D = 7$ diverges at two loops where the corresponding counterterm has a similar form to the $D = 4$ five-loop invariant. The occurrence of this divergence indicates that an off-shell version of the theory with six four-dimensional supersymmetries should exist, whereas if one could preserve seven such supersymmetries this divergence would not be allowed in $D = 7$. Unless there is a completely different mechanism at work in four dimensions, it therefore seems that the $N = 8$ theory is most likely to diverge at five loops.

3 Invariants in string theory and M-theory

In discussing invariants corresponding to field-theoretic counterterms it is enough to consider the linearised theory. In string theory or M-theory, however, we are more interested in the full non-linear expressions, and these are very difficult to find. Information about particular terms in full invariants has been derived from string scattering amplitudes and sigma model calculations while some other hints have been obtained using supersymmetry and arguments based on duality symmetries. An important point to note is that the linearised invariants of the type discussed in the preceding section cannot be generalised to the non-linear case in any straightforward manner. This is because the superspace measures do not exist in the interacting theory. For example, $D = 4, N = 8$ supergravity does not admit an $(8, 4, 4)$ harmonic superspace interpretation in the full theory.

Invariants of the R^4 type are reasonably well understood in $D = 10, N = 1$ supersymmetry where complete expressions are known for the bosonic terms [19]. There are two types of supergravity invariant. The first is a full superspace integral of the form [20]

$$\int d^{10}x\, d^{16}\theta\, E f(\phi) \sim \int d^{10}x\, e(\frac{d^4 f}{d\phi^4} R^4 + \ldots) \tag{9}$$

where E and e denote the standard densities in superspace and spacetime respectively, ϕ is the dilaton superfield, and where the R^4 terms appear in the combination $(t_8 t_8 - \frac{1}{8}\epsilon_{10}\epsilon_{10})R^4$ in the notation of, for example, Peeters *et al* [27], where these tensorial structures are explained. The second type of invariant can be constructed starting from certain Chern-Simons terms and so we shall refer to them as CS invariants. In the next section we shall see how such invariants can be explicitly constructed. There are two possible Chern-Simons terms in supergravity, $B \wedge \mathrm{tr}R^4$ and $B \wedge (\mathrm{tr}R^2)^2$. Details of all of these invariants up to quadratic order in fermions have been computed [19].

The structures associated with the $N = 1$ invariants are seen in various combinations in the $N = 2$ invariants. In IIA string theory the pure curvature term in the tree level invariant has the form $e^{-2\phi}(t_8 t_8 - \frac{1}{8}\epsilon_{10}\epsilon_{10})R^4$, which resembles the first type of $N = 1$ invariant, while there is also a one-loop term, required to cancel the five-brane anomaly, which arises from the CS term $B \wedge X_8$, where $X_8 = \mathrm{tr}R^4 - \frac{1}{4}(\mathrm{tr}R^2)^2$. This is believed to be associated with a pure R^4 term of

the form $(t_8 t_8 + \frac{1}{8}\epsilon_{10}\epsilon_{10})R^4$. The t_8^2 terms have been computed from string scattering [21, 22, 23], the tree-level ϵ^2 term can be inferred from sigma model calculations [24], the one-loop ϵ^2 term is suggested by one-loop four-point amplitudes [25], and the one-loop CS term is required for anomaly cancellation [26].

In IIB the linearised theory is described by a chiral superfield Φ and there is a linearised invariant of the form $\int d^{16}\theta\, \Phi^4 \sim (t_8 t_8 - \frac{1}{8}\epsilon_{10}\epsilon_{10})R^4 + \ldots$. This does not generalise to the full theory in any straightforward manner [2], however, due to the structure of the superspace constraints of the full theory [29]. Nevertheless, with the use of component supersymmetry, input from D-instanton results and $SL(2,\mathbb{Z})$ invariance some conjectures have been made about the scalar structure of some terms in the invariant. Specifically, each term appears multiplied by a function of the complex scalars $(\tau, \bar{\tau})$ which is a modular form under $SL(2,\mathbb{Z})$ whose weight is determined by the $U(1)$ charge of the term under consideration [30].

The R^4 invariant in M-theory has been discussed by various groups using string theory [31, 28], quantum superparticles [32] and supergravity [33, 34]. A straightforward supersymmetry approach is rendered even more difficult by the fact that the field strength superfield W has dimension one so that one would require an integral over eight odd coordinates of W^4 to obtain R^4. One result that is known is that there must be a CS term to cancel the anomaly of the M-theory five-brane [35]. This CS term has the form $C_3 \wedge X_8$, where C_3 is the three-form gauge field in the theory.

An alternative approach is to investigate the modified equations of motion rather than trying to compute the invariant directly. Corrections to the equations of motion can be understood as deformations of the on-shell superspace constraints, and the consistency conditions that the Bianchi identities place on these constraints lead, when one takes the into account the possibility of field redefinitions of the underlying superspace potentials, to a reformulation of the problem in terms of certain superspace cohomology groups. This cohomology, called spinorial cohomology, has been studied for various theories in the literature including M-theory considered as a deformation of $D = 11$ supergravity [36, 7]. In certain circumstances spinorial cohomology coincides with pure spinor cohomology which has been used to give a new spacetime supersymmetric formulation of superstring theories [37].

In the final section we shall investigate M-theoretic R^4 terms in a superspace setting by looking at the Bianchi identities in the presence of the five-brane anomaly cancelling term. It will be argued that the CS term gives rise to a unique invariant which is both supersymmetric and consistent with the anomaly and it will be shown how this invariant can be constructed. The deformations of the superspace constraints are driven by the anomaly term and can be found systematically, at least in principle.

4 M-theory in superspace

The component fields of $D = 11$ supergravity are the elfbein $e_m{}^a$, the gravitino, $\psi_m{}^\alpha$, and the three-form potential c_{mnp} [38]. In the superspace formulation of the theory [39] the first

[2] For an explicit attempt to do this see [28].

two appear as components of the supervielbein $E_M{}^A$, whereas the third is a component of a superspace three-form potential C_{MNP}.[3] We have

$$
\begin{aligned}
E^a = dz^M E_M{}^a \quad &= \quad \begin{cases} dx^m E_m{}^a & = dx^m(e_m{}^a + O(\theta)) \\ d\theta^\mu \, E_\mu{}^a & = d\theta^\mu(0 + O(\theta)) \end{cases} \\
E^\alpha = dz^M E_M{}^\alpha \quad &= \quad \begin{cases} dx^m E_m{}^\alpha & = dx^m(\psi_m{}^\alpha + O(\theta)) \\ d\theta^\mu \, E_\mu{}^\alpha & = d\theta^\mu(\delta_\mu{}^\alpha + O(\theta)) \end{cases}
\end{aligned}
\tag{10}
$$

while $C_{mnp}(x, \theta = 0) = c_{mnp}$.

The structure group is taken to be the Lorentz group, acting through the vector and spinor representations in the even and odd sectors respectively. We also introduce a connection one-form $\Omega_A{}^B$ which takes its values in the Lie algebra of the Lorentz group and define the torsion and curvature in the usual way:

$$
\begin{aligned}
T^A \quad &= \quad DE^A := dE^A + E^B \Omega_B{}^A = \frac{1}{2} E^C E^B T_{BC}{}^A \\
R_A{}^B \quad &= \quad d\Omega_A{}^B + \Omega_A{}^C \Omega_C{}^B = \frac{1}{2} E^D E^C R_{CD,A}{}^B
\end{aligned}
\tag{11}
$$

From the definitions we have the Bianchi identities $DT^A = E^B R_B{}^A$ and $DT^A = 0$. The assumption that the structure group is the Lorentz group implies that $R_a{}^\beta = R_\alpha{}^b = 0$ while

$$
R_\alpha{}^\beta = \frac{1}{4} (\gamma^{ab})_\alpha{}^\beta R_{ab} \tag{12}
$$

The equations of motion of supergravity in superspace are implied by constraints on the torsion tensor. In fact, it is enough to set

$$
T_{\alpha\beta}{}^c = -i(\gamma^c)_{\alpha\beta} \tag{13}
$$

to obtain this result [40]. The only other components of the torsion which are non-zero are

$$
T_{a\beta}{}^\gamma = -\frac{1}{36} \left((\gamma^{bcd})_\beta{}^\gamma W_{abcd} + \frac{1}{8} (\gamma_{abcde})_\beta{}^\gamma W^{abcd} \right) \tag{14}
$$

and $T_{ab}{}^\gamma$ whose leading component can be identified as the gravitino field strength. Given these results one can construct a superspace four-form G_4 which is closed and whose only non-zero components are

$$
G_{\alpha\beta cd} = -i(\gamma_{cd})_{\alpha\beta}, \qquad G_{abcd} = W_{abcd} \tag{15}
$$

The only independent spacetime fields described by these constraints are the physical fields of supergravity; their field strengths are the independent components of the superfield W.

[3] Notation: latin (greek) indices are even (odd), capital indices run over both types; coordinate (tangent space) indices are taken from the middle (beginning) of the alphabet.

Instead of deducing the existence of a four-form G_4 we can include it from the beginning. In this case one can show that there is a stronger result, namely that imposing the constraint

$$G_{\alpha\beta\gamma\delta} = 0 \tag{16}$$

is sufficient to imply the supergravity equations of motion [36, 41, 7]. Either the geometrical approach or the four-form approach can be used as a starting point for investigating deformations using spinorial cohomology, but we shall choose yet another route by introducing a seven-form field strength G_7 as well.[7] We then have the coupled Bianchi identities

$$dG_4 = 0\,, \qquad dG_7 = \frac{1}{2}(G_4)^2 + \beta X_8 \tag{17}$$

where we have included the anomaly term and where β is a parameter of dimension ℓ^6.

We shall work to first order in β which means that we can use the supergravity equations of motion in computing X_8, in other words, X_8 is a known quantity. The Bianchi identities can be solved systematically and all of the components of G_4, G_7, T and R can be found. Note that the only component of any of these tensors which can be zero at order β is $G_{\alpha_1\ldots\alpha_7}$. However, this solution might not be unique as there could be solutions of the homogeneous equations (i.e. without the X_8 term). In principle this question could be tackled using spinorial cohomology but we shall study it indirectly by looking at the action.

We briefly describe how one can construct a superinvariant from any CS invariant. If we have a theory in D spacetime dimensions formulated in superspace an invariant can be constructed if we are given a closed superspace D-form L_D [42, 43]. This invariant is

$$I = \int d^D x\, \epsilon^{m_1\ldots m_D} L_{m_1\ldots m_D}(x, \theta = 0) \tag{18}$$

Under a superspace diffeomorphism generated by a vector field v

$$\delta L_D = \mathcal{L}_v L_D = d(\iota_v L_D) + \iota_v dL_D = d(\iota_v L_D)\ . \tag{19}$$

Identifying the $\theta = 0$ components of v with the spacetime diffeomorphism and local supersymmetry parameters we see from this equation that the above integral is indeed invariant.

In some situations, notably when we have CS terms available, we can easily construct such closed D-forms [44, 45]. Suppose there is a closed $(D+1)$-form $W_{D+1} = dZ_D$ where Z_D is a potential D-form which we are given explicitly. We can always write $W_{D+1} = dK_D$, where K_D is a globally defined D-form, because the cohomology of a real supermanifold is equal to that of its body and this is trivial in degree $D+1$ in D dimensions. If we set $L_D = K_D - Z_D$ then L_D is closed and hence gives a rise to a superinvariant using the construction described above. Any CS term gives rise to a superinvariant in this manner.

In M-theory the appropriate forms are

$$\begin{aligned} W_{12} &= \frac{1}{2}G_4^3 + 3\beta G_4 X^8 \\ Z_{11} &= C_3(\frac{1}{2}G_4^2 + 3\beta X_8) \end{aligned} \tag{20}$$

The invariant constructed from these forms will include the anomaly-cancelling CS term and superpartners which will be of R^4 type. The purely bosonic terms, aside from the CS term itself, come from $K_{a_1 \ldots a_{11}}$, and these are the most difficult to compute. The easiest term to calculate is the lowest non-vanishing term which is $K_{abc\delta_1 \ldots \delta_8}$. It gives rise to terms with eight gravitinos in the action.

To this invariant we could add any invariant coming from a closed form L_{11}. It has been argued on (purely algebraic) cohomological grounds (subject to some assumptions) that there are no such terms [7]. If this is the case we would conclude that the R^4 invariants in eleven dimensions come from CS terms. In principle there could be two of these, since we have both $\mathrm{tr}R^4$ and $(\mathrm{tr}R^2)^2$, but M-theoretic considerations imply that we have to choose the linear combination which appears in X_8.

Finding the explicit form of this invariant is an extremely difficult task. One could start either by solving the G_7 Bianchi identities, or one could try to find K_{11} directly by solving the equation $W_{12} = dK_{11}$. This is currently under investigation.

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